RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

B.A./B.SC. SECOND SEMESTER EXAMINATION, MAY 2012

FIRST YEAR

PHYSICS (Honours)

Date : 21/05/2012 Paper : II

Time : 11 am – 2 pm Full Marks : 75

[Use Separate Answer Books for each group]

Group - A

UNIT-I : (Mech)

(Answer **any three** questions)

1. a) A particle moves outward along a spiral on a plane. Its trajectory is given by $r = A\theta$, where A is a constant. θ increases in time according to $\theta = \frac{\alpha t^2}{2}$, where α is a constant. Find the angle θ, at which the radial acceleration is zero. [2]

   b) Obtain expression for the velocity and acceleration of a particle in cylindrical polar coordinates. [2]

   c) A particle of mass m is projected vertically upwards under gravity, with initial speed u, in a medium that offers a resistive force (– kmv), v being the instantaneous speed.
   Find (i) the time to reach the maximum height, H;
   and (ii) the maximum height H.
   Show also that when k is very small, H can be approximated by,
   $$H \simeq \frac{u^2}{2g} - \frac{ku^3}{3g^2}$$ [6]

2. a) A system of particles is moving in an arbitrary manner where the internal forces between any two particles are Newtonian in nature. Show that the external torque about the centre of mass (CM) is equal to the rate of change of angular momentum about the CM even when the CM is not an inertial frame. [4]

   b) A uniform rod of length L and mass M is pivoted at one end and held horizontally. It is allowed to fall from rest under gravity. Find its angular speed and angular acceleration. Find the kinetic energy of the system about the CM. [3+1]

   c) Use the rocket equation to find the rocket residual mass m (in terms of the initial mass m_0) at which the momentum of the rocket is a maximum, for a rocket of mass m_0 starting at rest in free space. The exhaust velocity is a constant, v_0. [2]

3. a) A particle of unit mass moves in an elliptic orbit under the action of the force $f(r) = -\frac{k}{r^2}$, directed towards a fixed centre s.

   i) If a is the semi-major axis of the ellipse, show that the total energy E of the particle is a constant, equal to $E = -\frac{k}{2a}$.

   ii) Hence obtain an expression for the speed of the particle at any r. [4]

   b) If the speed at the farthest point is $v_2 = \sqrt{\frac{k}{3a}}$, show that

   i) eccentricity $e = \frac{1}{2}$

   ii) the speed at the nearest point is $v_1 = 3v_2$

iii) the angular velocity at the end of the latus rectum is $\omega = \sqrt{\frac{64k}{27a^3}}$. [1+1+2]

c) If T is the time period of the orbit, prove that $v_1 v_2 = \left(\frac{2\pi a}{T}\right)^2$. [2]

4. a) Find the expressions for the radial and cross-radial components of acceleration for planar motion. [5]

b) Consider a particle that feels only an angular force of the form $3m\dot{r}\dot{\theta}$. Show that if it starts from $\dot{\theta} \neq 0$ and $\dot{r} > 0$ it reaches $r = \infty$ in a finite time. Motion of the particle is in a plane. [5]

5. a) Obtain the inertia tensor of a uniform square plate of side l and mass m, with any vertex O as origin of coordinate axes lying along the edges. [3]

b) Let the plate rotate about the diagonal, with angular velocity $\vec{w}$.

i) Find the moment of inertia about the diagonal

ii) Show that the angular momentum $\vec{L}$ about the diagonal is parallel to $\vec{w}$.

iii) Find also the kinetic energy of rotation. [4]

c) Use the result from 5(b) to identify the principal axes through o, and hence calculate the corresponding principal moments of inertia. [3]

UNIT-II : (GPM)

(Answer **any two** questions)

6. a) What do you mean by torsional rigidity? Show that a hollow shaft is stronger than a solid one of the same mass. [5]

b) Show that a shear is equivalent to a compression and an equal extension at right angles to each other. [5]

7. a) Calculate the gravitational potential due to a uniform circular disc at a point on its axis. [5]

b) Calculate the gravitational self energy of a sphere. [5]

8. a) Derive an expression for the excess pressure over a curved liquid surface. [4]

b) Obtain an expression for the saturated vapour pressure over a curved liquid surface. [4]

c) A vertical U-tube consisting of two limbs of internal radii r_1 and r_2 $(r_1 < r_2)$ is partially filled with water. Calculate the difference in the level of water in the two limbs. [2]

9. a) Two vessels of equal cross section α are joined near their bases by a horizontal narrow tube of length l and radius r. Initially the heights of liquid in the vessels be $3h$ and h above the connecting tube. Calculate the time taken for the difference in levels to become h. [coefficient of viscosity is η and density is d]

b) What do you mean by critical velocity for fluid flow? Write an expression for it in terms of measurable quantities. What is Torricelli's theorem? [5+2+3]

Group - B

(Answer **any two** questions from Question No. 10 to 12 and **any one** from Question No. 13 & 14)

10. a) Derive an expression for the velocity of transverse waves along a string under tension.
What is the ratio of velocity of transverse wave to the longitudinal wave in the string if it is stretched by 0.04%? [4+1]

b) For the vibration of a stretched string of length l fixed rigidly at two ends, the transverse displacement at a point x at time t is given by,

$y(x,t) = \sum_{n=1}^{\infty} \sin\frac{n\pi x}{l}\left(a_n \cos\frac{n\pi ct}{l} + b_n \sin\frac{n\pi ct}{l}\right)$, in usual notation. Show that the amplitude of the nth harmonic is proportional to $\frac{1}{n^2}$. Given that the displacement at the point of plucking is 'h' at t=0, and the initial velocity is zero all over the string. [5]

11. a) The equation of motion of a damped harmonic oscillator driven by a constant force F_0 is given by $m\frac{d^2x}{dt^2} + R\frac{dx}{dt} + sx = F_0$, where m is the mass, R is the damping force per unit velocity and s is the restoring force per unit displacement.
Solve the equation for small damping and sketch graphically the nature of variation of x with time t. [6]

b) Using the principle of conservation of energy establish the differential equation of motion of a damped harmonic oscillator. [4]

12. a) Derive an expression for the velocity of a plane longitudinal wave in a fluid medium. Mention the assumptions you make. [5+2]

b) Can the speed of sound in air exceed the r.m.s. speed of air molecules due to thermal agitation? Discuss it. [3]

13. Two sources are emitting light of same frequency with different amplitudes. Derive the expression for the intensity distribution when these lights are superposed. Hence discuss the necessity of coherent sources to produce sustained interference pattern. [3+2]

14. a) What is phase velocity and group velocity? Does the shape of a wave group change in a dispersive medium? [3]

b) Find the wavelength of sound waves of frequency 200 Hz produced in hydrogen.
Given: Velocity of sound in air = 332 m/s
Density of air is 14.4 times greater than that of hydrogen. [2]
